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THE DETERMINATION OF THE CONSTANTS OF INSTRUMENT TRANSFORMERS.[1]

By P. G. Agnew and T. T. Fitch.

The determination of the ratio and phase angle of instrument transformers is an important problem, and one in which considerable precision is required for technical work. The principle of the method here described is the same for both current and potential transformers, being an application of the potentiometer method.

POTENTIAL TRANSFORMERS.

If a high noninductive resistance be connected in parallel with the primary, the secondary voltage may then be applied to it potentiometer fashion, as shown in Fig. 1, by connecting one ter-

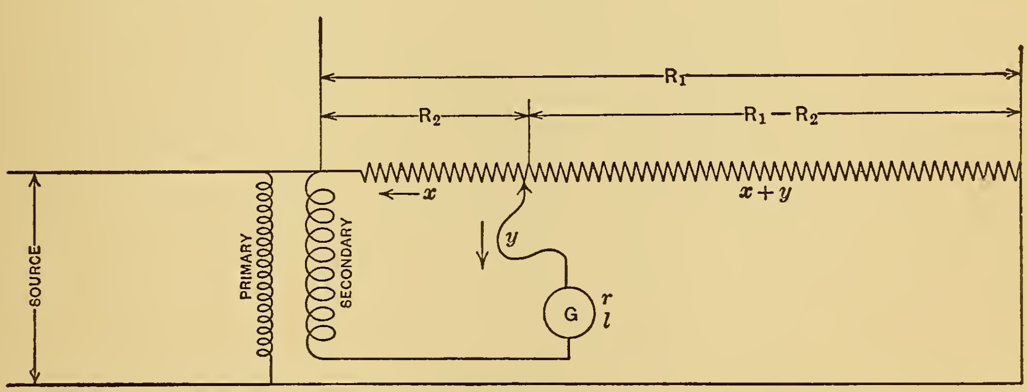

Fig. 1.

minal in a reversed sense to one end of the resistance and applying the other terminal, through a detector G, to the resistance. If there were no phase difference between the primary and secondary voltages, a point could be found at which no current would flow

[1] Paper read before the Washington meeting of the American Physical Society, April 27, 1909. A preliminary account of the work appeared in the Electrical World.

through the detector. But, in general, there is a phase difference, so that the emf. at the terminals of G can only be reduced to a minimum value Q, Fig. 2, which is in quadrature with the primary

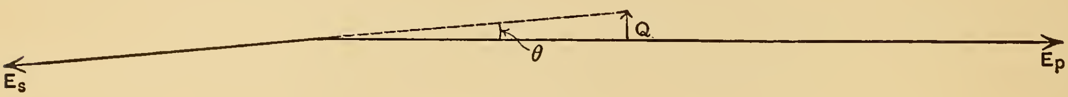

Fig. 2.

emf. If, then, we had an idiostatic voltmeter that would read to 0.001 volt with an accuracy of 1 or 2 per cent, we should have to adjust for a minimum only and then read the value of Q, the ratio of the total resistance to the part tapped off giving the voltage ratio of the transformer and the ratio of Q to E_2 giving tan θ. But as such an instrument does not seem to be available, some indirect method has to be used. The method first tried was to balance the small quadrature component Q, using as a source of emf. a second phase from a two-phase machine, and making use of a telephone as detector. The capacity effects of the various parts of the circuit reduced the accuracy at low frequencies on account of the greater sensitiveness of the telephone to the harmonics present. Either a thermo-galvanometer or a vibration galvanometer gave greater accuracy, but neither was satisfactory as regards ease of manipulation. A method considerably more accurate is to use a vibration galvanometer and only a single source of emf., placing variable inductances in the two parts of the high resistance circuit and alternately adjusting the resistances and inductances for zero deflection, and then calculating the difference in the phase angles by which the emf. leads the current in the two parts of the high resistance.

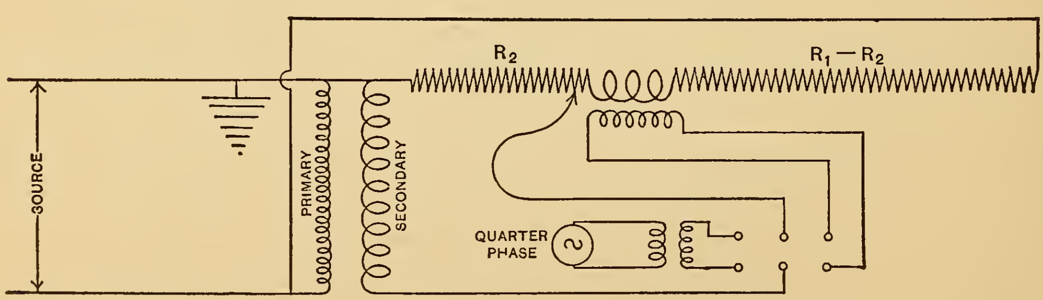

Fig. 3.—*Two-dynamometer method for test of potential transformers.*

The method finally adopted as most satisfactory was to use as detectors two electrodynamometers (Fig. 3). The field coil of one

was placed in series with the high resistance, while the field of the second was energized by a current in quadrature from a two-phase machine. The operation is to throw the switch to the right and adjust R_2 for no deflection. The current in the moving coil is then not necessarily zero, but is in quadrature with that in R_1. Then throw the switch left and read the deflection which measures the quadrature component Q. The first setting gives the ratio

$$\frac{E_1}{E_2}=\frac{R_1}{R_2} \tag{1}$$

while the deflection of the second dynamometer gives the phase angle [2]

$$\tan\theta=\frac{Q}{E_2}$$

The latter instrument draws a very small current from the network of conductors, and hence for the phase angle measurement has to be calibrated as a voltmeter under the conditions of use. At first sight it would seem that its multiplier should be R_2 only (Fig. 3). But it will be shown by analysis, as seems evident from a close study of the figure, that R_1-R_2 is in parallel with R_2, so that for calibration they are so connected.

Also if the time constant of R_2 differs appreciably from that of the total R_1, then the emf. at its terminals will not be in phase with E_1, and a correction equal to the phase difference will have to be applied to the value of the phase angle obtained by deflection.

In order to establish these results analytically, and at the same time to investigate the magnitude of the possible corrections due to residual inductances, let us assume the phase angle of the reversed secondary voltage is positive if it leads the primary. This accords with the customary geometrical convention in dealing with complex quantities. Then in Fig. 1, let

R_1 = total resistance in the primary circuit.

R_2 = resistance included in the secondary.

r = resistance of the detecting dynamometer.

L_1, L_2, l = corresponding inductances.

E_2+iQ = secondary emf., E_2 being the part in phase with E_1, Q the part in quadrature.

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[2] Mr. L. T. Robinson, in the Proc. A. I. E. E., July, 1909, p. 981, describes a method involving the same potentiometer principle, but the component in quadrature is rectified by a rotating commutator and measured with a direct current instrument.

θ = phase angle.
x, y = vector currents in the directions shown.
$p = 2\pi$ times the frequency.
$i = \sqrt{-1}$.

Then applying Kirchhoff's law to the two circuits

$$E_1 = x(R_2 + ipL_2) + (x+y)[R_1 - R_2 + ip(L_1 - L_2)]$$
$$= x(R_1 + ipL_1) + y[R_1 - R_2 + ip(L_1 - L_2)]$$

and

$$E_2 + iQ = x(R_2 + ipL_2) - y(r + ipl).$$

Solving the last two equations for y,

$$y = \frac{\begin{vmatrix} R_1 + ipL_1 & E_1 \\ R_2 + ipL_2 & E_2 + iQ \end{vmatrix}}{\begin{vmatrix} R_1 + ipL_1 & R_1 - R_2 + ipL_1 - ipL_2 \\ R_2 + ipL_2 & -r - ipl \end{vmatrix}}.$$

In expanding the determinant, since L_1, L_2, and l are small quantities, we may neglect terms containing their squares or products, whence

$$y = \frac{E_2R_1 - E_1R_2 - QpL_1 + i(QR_1 + E_2pL_1 - E_1pL_2)}{-R_1r - R_1R_2 + R_2^2 + i(-R_1pl - rpL_1 - R_2pL_1 + 2R_2pL_2 - R_1pL_2)}.$$

Rationalizing the denominator, and again neglecting squares and products of small quantities,

$$y = \frac{(E_2R_1 - E_1R_2 - QpL_1)}{-R_1r - R_1R_2 + R_2^2}$$
$$- i\frac{(E_2R_1 - E_1R_2)(-R_1pl - rpL_1 - R_2pL_1 + 2R_2pL_2 - R_1pL_2)}{(-R_1r - R_1R_2 + R_2^2)^2}$$
$$+ i\frac{(-R_1r - R_1R_2 + R_2^2)(QR_1 + E_2pL_1 - E_1pL_2)}{(-R_1r - R_1R_2 + R_2^2)^2} \qquad (2)$$

Since the current through the dynamometer (see Fig. 3) is only very slightly out of phase with E_1, we may, for the condition of zero deflection in the series dynamometer, set the real part of y equal to zero.

$$E_2R_1 - E_1R_2 - QpL_1 = 0$$

Dividing by E_2R_2

$$\frac{E_1}{E_2} = \frac{R_1}{R_2} - \frac{QpL_1}{E_2R_2}$$

The last term is an extremely small correction to the ratio of the resistances, and is entirely inappreciable, amounting to less than one part in 50,000 in the most unfavorable case examined.[3] Strictly, the ratio is

$$\frac{E_1}{\sqrt{E_2^2+Q^2}}=\frac{R_1}{R_2}\cos\theta$$

but as the phase angle of a good potential transformer designed for use with instruments will always be less than 30′, and as the cosine of 30′ differs from unity by only one part in 25,000, we may take as the ratio

$$\frac{E_1}{E_2}=\frac{R_1}{R_2} \qquad \text{(1) bis}$$

Q, which gives a measure of the phase angle, may be determined from the quadrature component of y in equation (2). It is to be noticed that the numerator of the second term of (2) reduces to zero, since for the ratio balance

$$E_2R_1-E_1R_2=0$$

Hence

$$y_{(i)}=\frac{QR_1+E_2pL_1-E_1pL_2}{-R_1r-R_1R_2+R_2^2}$$

$$\therefore Q=-y_{(i)}\left(r+R_2-\frac{R^2}{R_1}\right)-\frac{E_2pL_1-E_1pL_2}{R_1}$$

$$\therefore \theta=\tan\theta=\frac{Q}{E_2}=-\frac{y_{(i)}}{E_2}\left(r+R_2-\frac{R_2^2}{R_1}\right)+\left(\frac{pL_2}{R_2}-\frac{pL_1}{R_1}\right) \qquad (4)$$

The expression $\left(\frac{r+R_2-R_2^2}{R_1}\right)$ is the effective resistance of the moving coil circuit, the two parts of the high resistance R_2 and R_1-R_2 being in parallel, but the whole in series with r.

Hence the numerator of the first term of (4) is the emf. measured by the quarter phase dynamometer. The last term is a constant correction depending on the difference in the time constants of the two parts of the high resistance R_1. With high grade resistances this correction may easily be made less than one minute at 60 cycles.

[3] If R^1, R^2, and r have the same time constants, there are no corrections to either ratio or phase angle.

The calibration is easily accomplished without the use of direct current by keeping a known resistance in series with the field of the quarter phase dynamometer, with a switch so that the emf. at its terminals may be thrown into the potential circuit, while the transformer is disconnected from the source. This introduces a known emf. in the moving coil circuit which gives the calibration as voltmeter. The two parts of the high resistance are thrown in parallel by short-circuiting both windings. In fact, since the resistance of the windings is nearly always negligible in comparison to the other parts of the circuit, and as the calibrating current divides between the two paths in such a manner as to neutralize their magnetizing effects upon the core, the deflection of the dynamometer is the same whether the windings are short-circuited or not.

This two-dynamometer method has the advantages of giving a uniform scale, practically independent adjustments for the two settings, a high sensibility, and the ratio can be determined at zero load on the secondary. The correction factor for any slight departure from exact quadrature of the field in the second dynamometer is inappreciable except for the most precise measurements, in which case the adjustment may either be made or easily corrected for.

With the instruments used in obtaining the ratio and phase angle curves shown below, about 0.05 ampere was drawn through R_1, and a constant current of 0.5 ampere maintained in the field of the quarter phase dynamometer. For the safety of the operator the point of connection of the two transformer windings is connected to earth, and the portion $R_1 - R_2$ put out of reach. (Fig 4.) This particular arrangement gave a maximum sensibility of one part in 20,000 in ratio, and about 0.1 minute in phase angle, but the absolute accuracy is of course less, being limited by the accuracy of the high resistances in the ratio determination and by the residual reactances in the multipliers in the determination of phase angles.

The method has been compared with the differential voltmeter method described by Rosa and Lloyd [4] for the ratio determina-

[4] This bulletin, **6**, p. 1, 1909.

tion, and with a modification of their method for the phase angle measurement, in which a quarter phase current from a machine was substituted for the condenser current used by them. The results agreed within the accuracy of these methods, as did also a second method of measuring phase angles, described by the same authors, in which the phase angle was read off directly as the angle by which a stator armature of a generator was shifted to give null settings on a dynamometer, whose field was excited by this generator and whose moving system was energized first by the primary, and then by the secondary of the transformer under test, the transformer being fed by a second generator rigid on the same shaft.

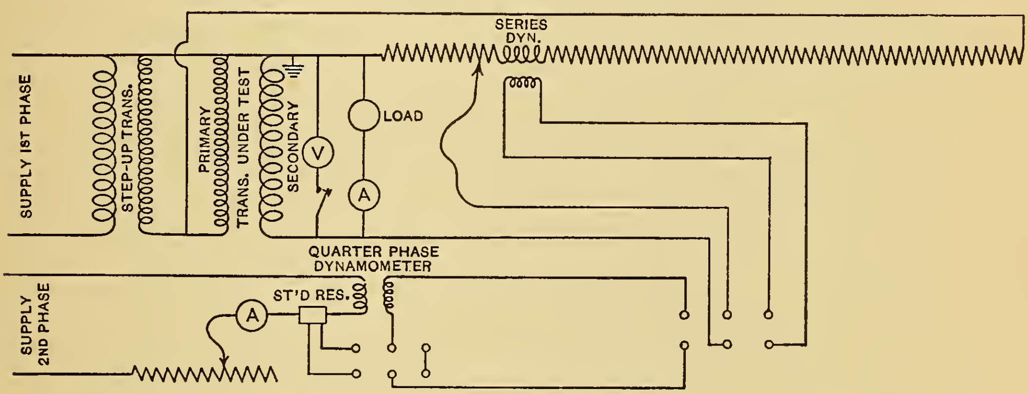


Fig. 4.—*Complete diagram of connections for test of potential transformer.*

A check not requiring the use of a dynamometer was furnished by the method mentioned above, of using a vibration galvanometer as detector, and placing variable inductances in the two parts of high resistance. This requires a simultaneous balance of resistance and inductance. It is fully as sensitive as the dynamometer method. The values of ratio given by the two methods agreed precisely, but the phase angles showed a discrepancy of about 0.4′. While this difference is too small to have any practical significance, it appeared as a constant difference, and was thought to be due to a charging current into the galvanometer.

In Figs. 5 to 9 are shown load curves of typical potential transformers, designed especially for use with measuring instruments, from different makers. The full-line curves are for noninductive load, while the isolated points shown in some instances are for the highly inductive load of the potential coil of a watthour meter,

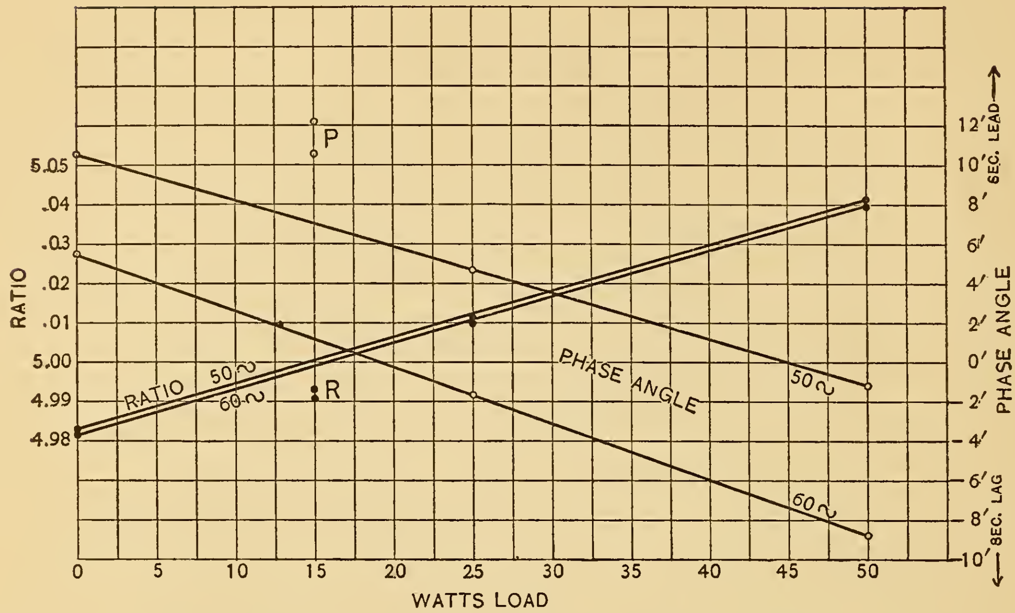


Fig. 5.—*Potential transformer M_1. Isolated points P and R are phase angle and ratio plotted in volt-amperes for inductive load, 20 per cent power factor.*

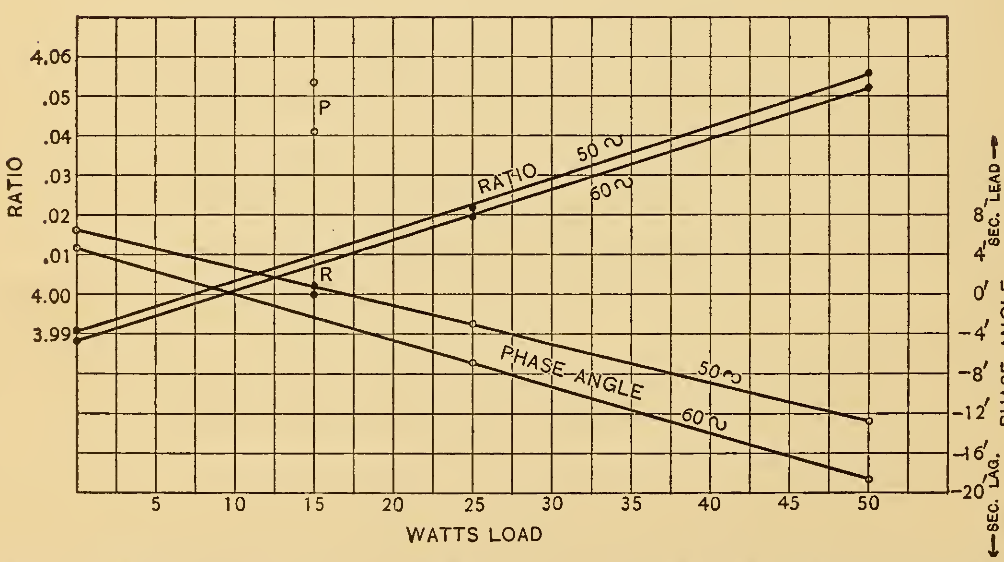


Fig. 6.—*Potential transformer M_2. Isolated points P and R are phase angle and ratio plotted in volt-amperes for inductive load, 20 per cent power factor.*

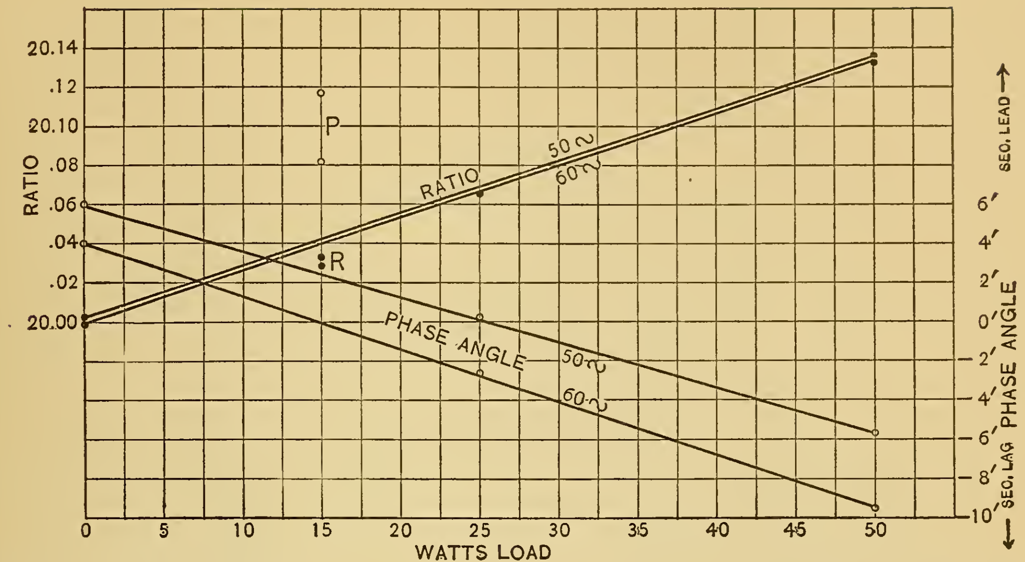


Fig. 7.—*Potential transformer N. Isolated points R and P are ratio and phase angle plotted in volt-amperes for 20 per cent power factor.*

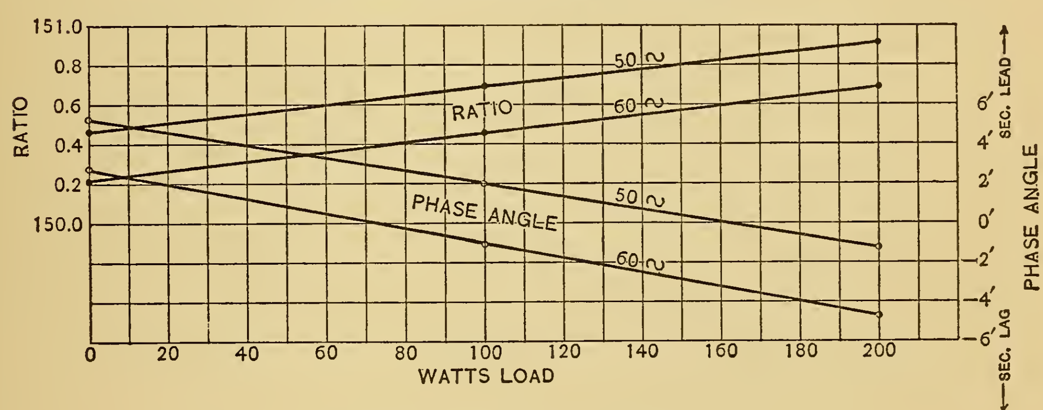


Fig. 8.—*Potential transformer O.*

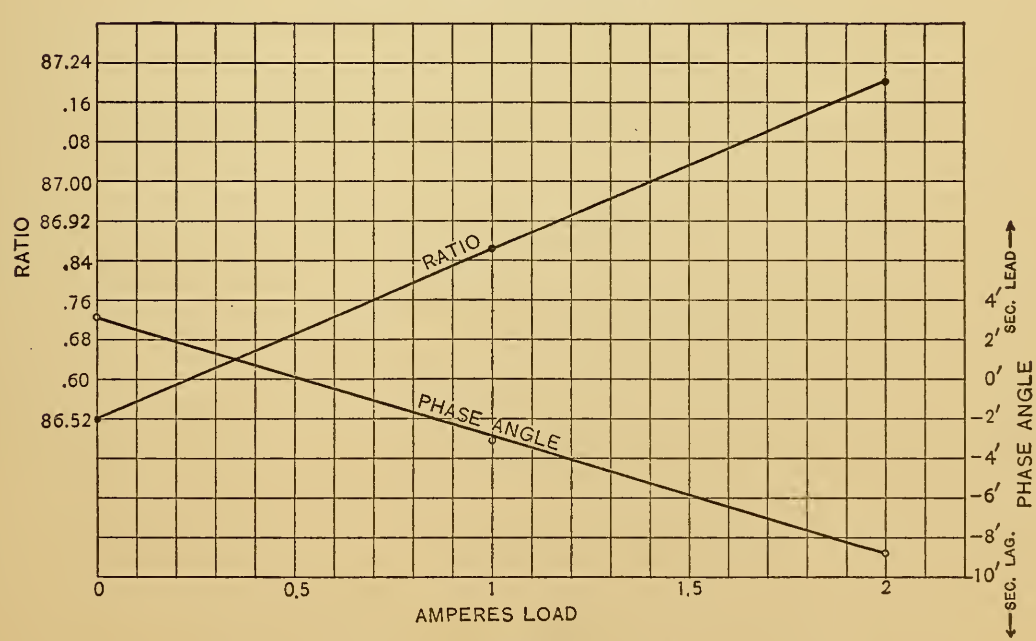


Fig. 9.—*Potential transformer P. Frequency 50 cycles.*

being plotted with voltamperes as abcissas for comparison. Descriptive data for these transformers is collected in Table I.

TABLE I.

Data for Potential Transformers.

Transformer	Voltage	Nominal ratio	Rated frequency	Watt rating	Resistance	
					Primary	Secondary
M_1	550	5/1	50	50	30.5	1.54
M_2	440	4/1	50	50	30.5	1.88
N	2,200	20/1	50	50	460	0.393
O	16,500	150/1	50	200	1,255	0.112
P	8,660	86.6/1	50	200	1,600	0.175

That a modern, well-designed potential transformer is an instrument of high precision may be seen from a study of these curves. The regulation of the better transformers is less than 1 per cent, and in some cases is but 0.3 per cent.

This refers to the nominal load ratings given by the maker, which are 50 watts for the smaller and 200 watts for the larger sizes. But if the transformer be used only in connection with a limited number of instruments the load will be but a few watts, and consequently the regulation for actual load is very much smaller. For example, the ratio of a small transformer will be changed less than 0.2 per cent by connecting a 150-volt voltmeter to the secondary, and the ratio of a large transformer would be changed only 0.02 per cent. For special purposes makers supply transformers rated as low as 10 watts, in which case the changes would be larger.

Both the ratio and the phase angle curves for constant voltage are strictly linear. The phase angle is so small that for all ordinary technical uses it may be neglected, as it brings in an appreciable error only at extremely low-power factors, and even in this case the other instruments used in a power measurement will usually limit the accuracy rather than the potential transformer.

Fig. 10 shows the effect of variation of voltage upon ratio and phase angle. From this and from the other curves it is seen that the ratio is affected only to a very slight degree by considerable variations in voltage and frequency. In case the core is worked at too great a flux density, so that the magnetizing current becomes excessive, the effect of change of voltage or of frequency may become appreciable. This was the case with transformer O, which took a magnetizing current more than three times as great as is customary for transformers of like range.

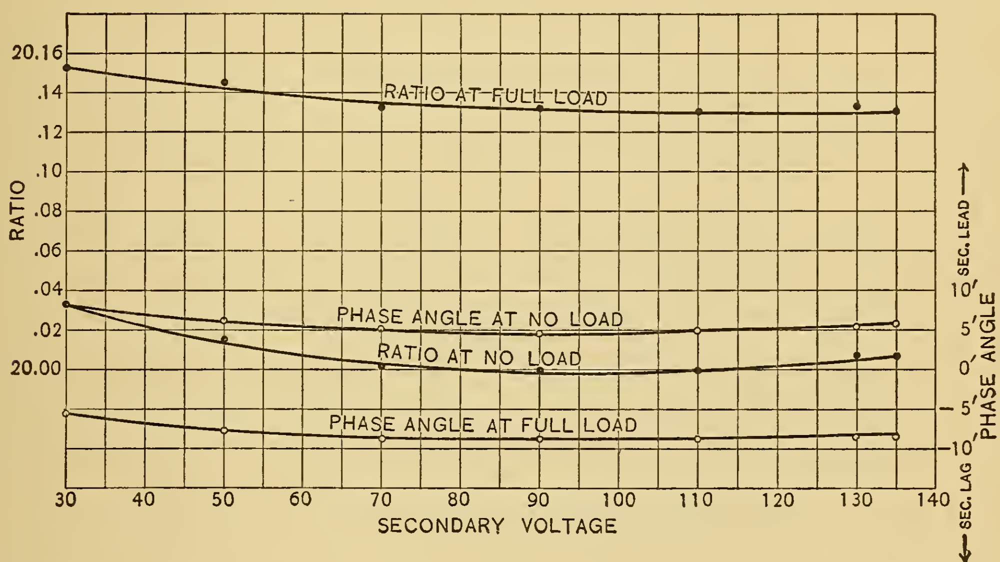


Fig. 10.—*Showing effect of change of voltage upon ratio and phase angle. Potential transformer N.*

DIFFERENTIAL TEST.

The same method may be used to determine differences in ratio and in phase angle between the transformer under test and a second one used as a standard by putting their primaries in parallel on the same source and measuring the ratio of their secondary voltages and the phase angle between them. This avoids the necessity of working on the high-voltage side. The sensibility goes up enormously, as the resistance R_1–R_2 is here reduced to a few ohms, thus lowering the resistance in series with the moving coil of the dynamometer. With the same instruments the sensibility, when used differentially, was a part in a million in ratio, and 0.05″ in phase angle, though, of course, the *absolute* accuracy is not at all increased. In fact, by using the pivoted movements of portable dynamometer voltmeters, sensibility greater than is

needed in any technical work may be obtained. Hence the use of standard transformers whose ratio and phase angle are to be determined once for all is proposed.

This method could easily be made to read differences in ratio directly in hundredths of a per cent and differences of phase angle directly in minutes. By bringing out several secondary taps a single transformer could be made to cover a considerable range. For example, if a 60,000-volt transformer had its primary divided in four sections, it could be connected for 60,000, 30,000, or 15,000 volts, and by determining its voltage-ratio curve for different frequencies, it could be used as a standard for testing transformers down to three or four thousand volts. Its ratio and phase angle would have to be determined for but one resistance load on its secondary.

Experience has shown that carefully designed transformers having series-parallel connections have the same phase angle, no matter what the grouping, and that factors connecting the different ratios are very accurately 2 and 4. Hence such a transformer could be tested to 15,000 volts and the ratio and phase angle assumed for the higher voltage connections.

CURRENT TRANSFORMERS.

The problem of the current transformer has been a more difficult one, for the resistance and inductance of a sensitive instrument placed in the secondary circuit will change both ratio and phase

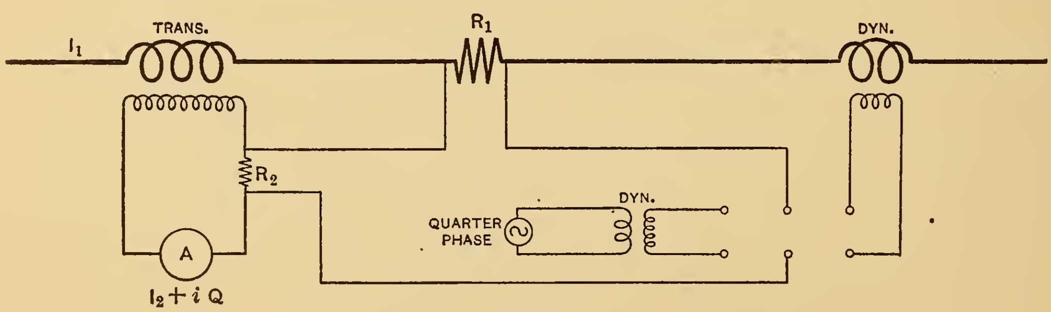


Fig. 11.

angle very appreciably, so that results thus obtained do not correspond to the conditions of use. But the same potentiometer principle used in the case of the potential transformer may be applied. An electrodynamometer and a low noninductive resistance R_1 (Fig. 11) are placed in series with the primary of the

transformer, and a second noninductive resistance, R_2, of such a value that R_2/R_1 is approximately the ratio of transformation, is placed in the secondary circuit. The potential terminals of these resistances are joined so as to oppose the emfs. impressed upon them, the leads being brought to a double throw switch so that the resultant emf. may be applied to the moving coil of either the series or of the quarter phase instrument. The switch is thrown to the right and R_2 adjusted (by shunting) to give no deflection. This gives the ratio of the currents. The switch is then thrown to the left and the component of emf. in quadrature, Q, measured by deflection. This gives the phase angle.

In developing the relations we shall find that a correction must be applied to the phase angle measurement due to the difference in the phase angles of the shunts. As in the case of the potential transformer, let

R_1, R_2, r = the resistances of the shunts in the primary and secondary, and in the moving coil circuit, respectively.

L_1, L_2, l = the corresponding inductances.

I_1, I_2, x = the corresponding currents.

θ = the phase angle. Then

$$I_1(R_1+ipL_1)-(I_2+iQ)(R_2+ipL_2)=x(r+ipl)$$

$$x=\frac{I_1R_1+iI_1pL_1-I_2R_2-iI_2pL_2-iQR_2+QpL_2}{r+ipl}$$

or, neglecting terms containing products or squares of L_1, L_2, and l,

$$x=\frac{I_1R_1r-I_2R_2r+QrpL_2-QR_2pl}{r^2+p^2l^2}$$

$$+\frac{i(I_1r\,pL_1-I_2rpL_2-QR_2r-I_1R_1pl+I_2R_2pl)}{r^2+p^2l^2} \tag{5}$$

For the balance for the determination of ratio, the real part of (5) becomes zero.

$$I_1R_1r-I_2R_2r+QrpL_2-QR_2pl=0$$

$$\frac{I_1}{I_2}=\frac{R_2}{R_1}+\frac{Qp}{I_2R_1r}(R_2l-L_2r)=\frac{R_2}{R_1}\left[1+\frac{Q}{I_2}\left(\frac{pl}{r}-\frac{pL_2}{R_2}\right)\right]$$

The last term, like the corresponding one in (3) for potential transformers, is negligible, being less than one part in 5,000 at full load in the most unfavorable case examined.[5] Hence we may take as the ratio of the currents in phase

[5] If R_1, R_2, and r have the same time constants, there are no corrections to either ratio or phase angle.

$$\frac{I_1}{I_2}=\frac{R_2}{R_1}$$

and for the actual ratio, taking account of phase angle

$$\frac{I_1}{\sqrt{I_2^2+Q^2}}=\frac{R_2}{R_1}\cos\theta \tag{6}$$

For most work the correction due to the cosine factor may be neglected, as it amounts to but a part in 6,000 for a phase angle of 1°, and is but a part in 1,000 at 2.5°, and the phase is large only at light loads where the accuracy needed is not as great.

In the measurement of Q, the deflection is due entirely to the quadrature component of x.

From (5)

$$x_{(i)}=\frac{(I_1rpL_1-I_2rpL_2)-QR_2r-(I_1R_1pl-I_2R_2pl)}{r^2+p^2l^2}$$

Since for the ratio setting $(I_1R_1-I_2R_2)$ is made zero, we have

$$x_{(i)}=\frac{pr(I_1L_1-I_2L_2)-QR_2r}{r^2+p^2l^2}$$

From which, if we neglect p^2l^2 in comparison with r^2,

$$Q=\frac{1}{R_2}(-x_{(i)}r)+\frac{p(I_1L_1-I_2L_2)}{R_2}$$

$$\therefore\ \theta=\tan\theta=\frac{Q}{I_2}=\frac{-x_{(i)}r+p(I_1L_1-I_2L_2)}{I_2R_2}$$

Now, since $I_2R_2=I_1R_1$, we have, finally,

$$\theta=\frac{-x_{(i)}r}{I_2R_2}+\left(\frac{pL_1}{R_1}-\frac{pL_2}{R_2}\right) \tag{7}$$

The numerator of the first term represents the emf. measured by the quarter phase dynamometer, while the expression in the parenthesis represents a constant correction due to difference in the time constants of the two shunts.

Thus for the most accurate work, constant corrections have to be applied to the measured phase angle both in the case of the potential transformer (equation 4) and in the case of the current transformer (equation 7). With the assumptions made above in regard to signs, these corrections, when positive, increase the angle of secondary load over the value that would be calculated by the deflection alone.

It should be noted that the negative sign before the first term of the right members of equations 4 and 7 is without significance so far as giving a sign to an experimental value is concerned, since it would require the interpretation of the direction of a dynamometer deflection. In order to determine whether a given deflection means a lagging or a leading secondary voltage in the case of a potential transformer, we have only to note whether a small noninductive load added to the secondary increases or decreases the deflection, since a noninductive load always tends to rotate the vector representing the secondary emf. backward. Normally, θ starts at no load with a small positive value (secondary leading) and changes to a negative value for full-rated noninductive load. An inductive load makes the secondary lead still more.

In the case of the current transformer the secondary current usually leads, θ becoming negative only for very inductive loads. The angle of lead is always greater for a small current than for full-load current.

For the most accurate work the moving coil of the series dynamometer should be set at the point of zero mutual inductance, and that of the quarter phase dynamometer worked as near the point of zero mutual inductance as practicable. Also, considerable care must be taken to keep the leads twisted together to avoid induction from stray fields.

With the dynamometers used, a maximum sensibility, in the case of good transformers at full-load current, of about one part in 20,000 in ratio and $0.1'$ in phase angle was obtained by using from 0.1 to 0.4 volt drop in the shunts, but of course the final accuracy is less. This requires but a few hundredths of an ohm to be placed in the 5-ampere secondary of the transformer, which is comparable to the resistance of connecting leads as ordinarily used. The energy necessary to operate the instruments is taken chiefly from the primary instead of from the secondary.

As the sensibility in making the ratio settings varies inversely as the square of the current, dynamometers having two current ranges were ordinarily used in order to keep up the sensibility at low loads. The sensibility of the measurement of phase angle varies inversely with the load.

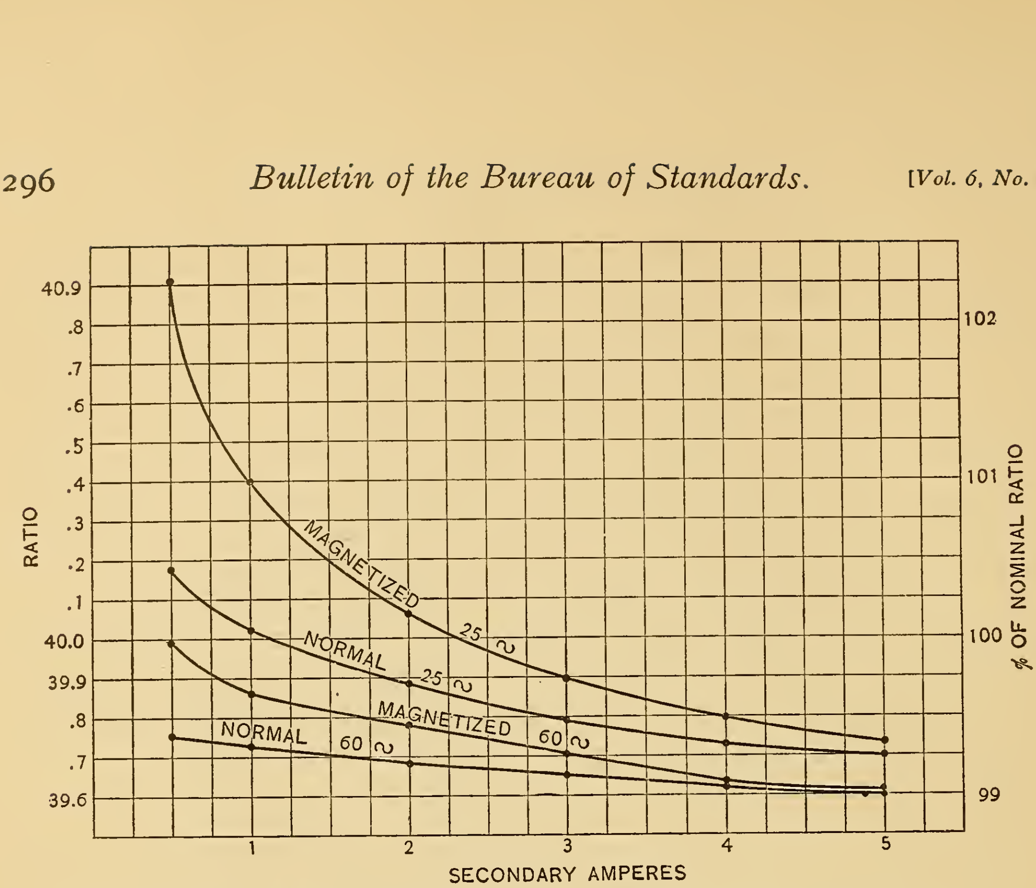


Fig. 26.—*Showing effect of magnetic history on the ratio of current transformer A.*

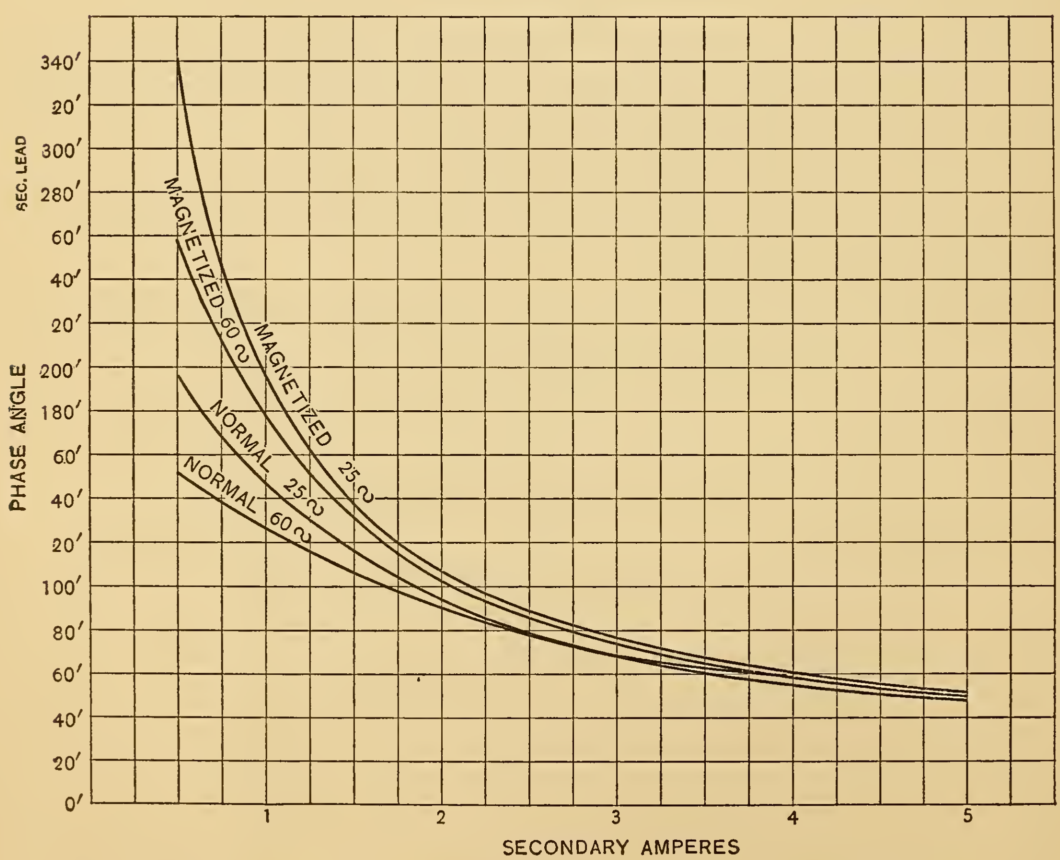


Fig. 27.—*Showing effect of magnetic history on phase angle, current transformer.*

The multiplier of the quarter phase instrument was usually adjusted to give a deflection of 1 centimeter per millivolt, and the calibration was made by introducing the emf. at the terminals of a resistance of 0.01 ohm placed in series with the field coil, as explained above.

In Figs. 12 to 25 are shown ratio and phase angle curves for different types of secondary loading. For convenience of reference both the actual ratio and the ratio expressed in percentage of nominal ratio are given on the vertical scale.

TABLE II.

Data for Current Transformers.

Trans-former	Primary current	Nominal ratio	Rated frequency	Watt rating	Working voltage	Secondary resistance
A	200	40/1	25–125	40	15,000	0.463
B	200	40/1	25	....	12,000	0.353
C	10	1/1	25	....	220	0.050
D	250	50/1	25–100	....	12,000	0.204
E	200	40/1	25–125	40	15,000	0.70

EFFECT OF MAGNETIC HISTORY.

It does not seem to have been before noted that both the ratio and the phase angle of a current transformer may be changed by the past magnetic treatment it has received.

If a direct current of full-load value be passed through either winding, the flux rises to an abnormally high value, and when the circuit is broken the iron does not return to a neutral state, but is left in a state in which its differential permeability is less. Hence, when it is again used to measure an alternating current, it draws a higher magnetizing current than before, and both its ratio (primary to secondary current) and its phase angle are increased. The effect is precisely the same if the secondary be accidentally open-circuited while alternating current is passing through the primary, although the amount by which the calibration curves will be shifted depends upon the point in the cycle at which the circuit happens to be closed. Figs. 26 and 27 show the effect of such treatment in raising the entire curves, both of ratio and of phase angle, in typical cases.

This effect has been found to be very appreciable in every transformer investigated, and is also present with every condition of secondary load. As indicated by theory, it is greater the greater the impedance in the secondary circuit, and decreases as the load increases. The change in ratio at full load is nearly always about 0.1 per cent, but at one-tenth load it may be anywhere from one-half to five or even ten per cent, depending upon the quality of the transformer core and the amount of resistance and inductance in the secondary.

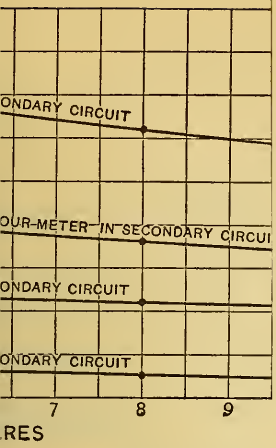
This is no doubt a frequent source of serious error in measurements requiring the use of current transformers. It is clear that the secondary of a current transformer should never be open-circuited while current is passing through the primary, and that the direct current used in calibrating standards, sometimes used with them, must not be allowed to pass through the transformer.

Fortunately, the effect may be entirely removed, in case the secondary has been accidentally open-circuited, by demagnetizing the core. This may be conveniently done by opening the secondary, passing full load, or an overload alternating current through the primary, and continuously reducing it to zero.

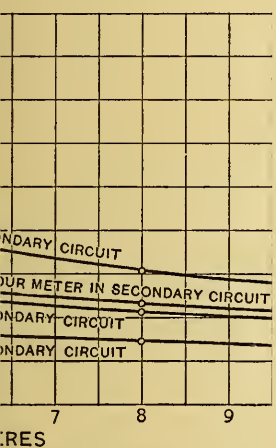
DETERMINATION OF THE INDUCTANCE OF SHUNTS.

The determination of the residual inductances of low resistance shunts presents great difficulties if attempted by ordinary bridge methods, as the inductance of the connecting leads is many times the inductance to be measured. For example, the inductance of a millimeter wire 1 centimeter long is 4.5×10^{-9} henry. A 0.001 ohm shunt having the same inductance will show a phase angle of 6′ at 60 cycles.

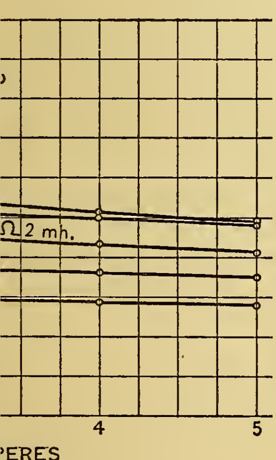
The quarter-phase dynamometer can be used to compare the inductances of two shunts, if a known current be passed through them in series (Fig. 28). If the inductance of one of them is known, and if a phase-shifting device is at hand, the comparison may be conveniently made by calculating the reactance emf. of the standard shunt at the given current and frequency, and then shifting the phase of the field of the dynamometer to give the calculated deflection when the shunt terminals are connected to the moving coil. This brings the two currents into exact quad-



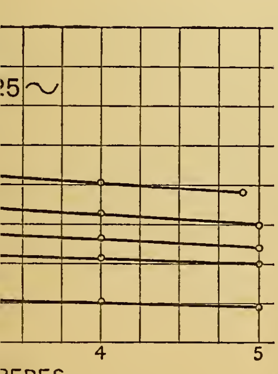
transformer C.



ent transformer C.



ormer D. 60 cycles.



ormer D. 25 cycles.

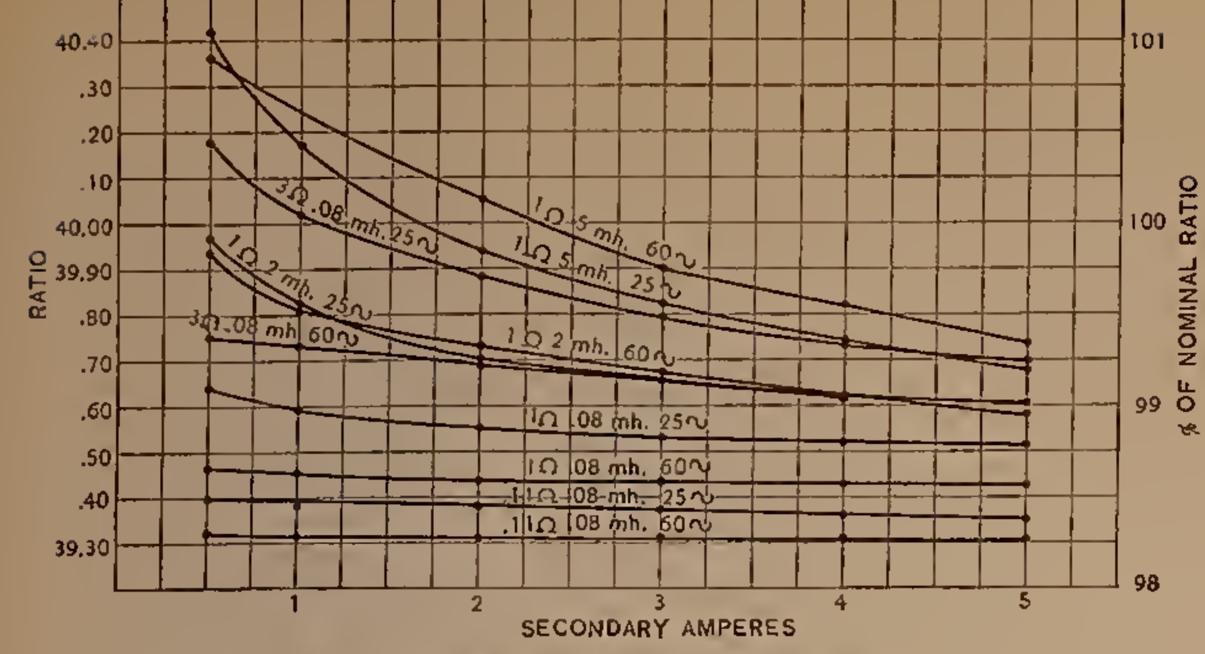


Fig. 12.—*Ratio curves, current transformer A.*

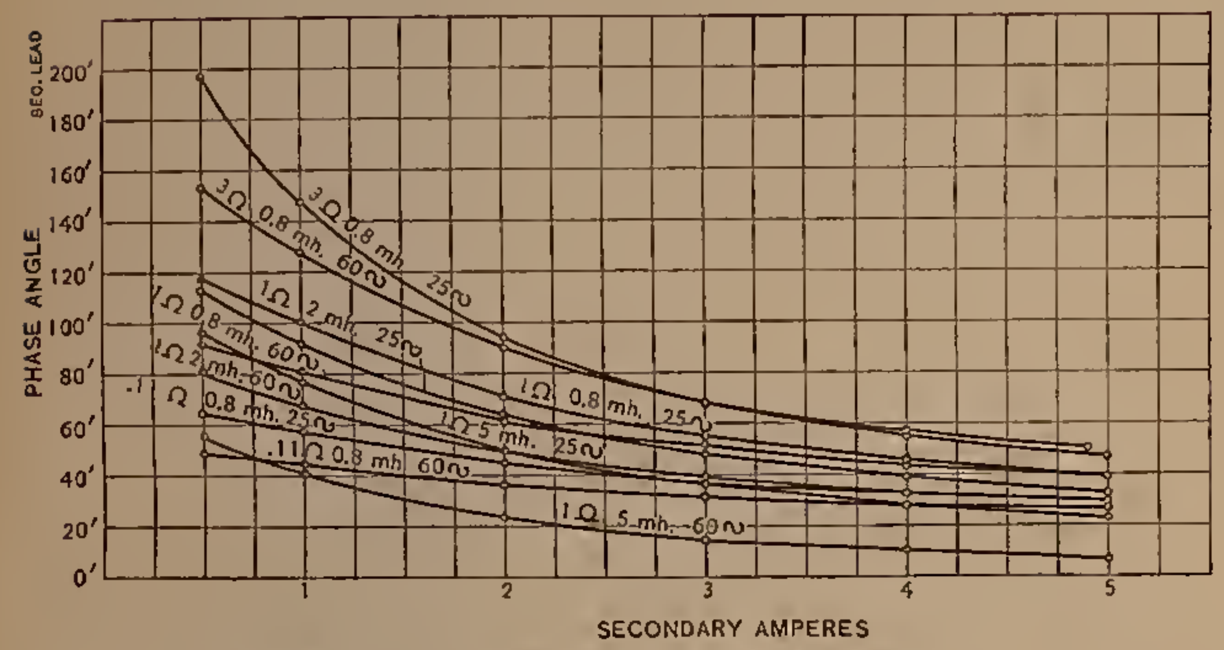


Fig. 13.—*Phase angle curves, current transformer A.*

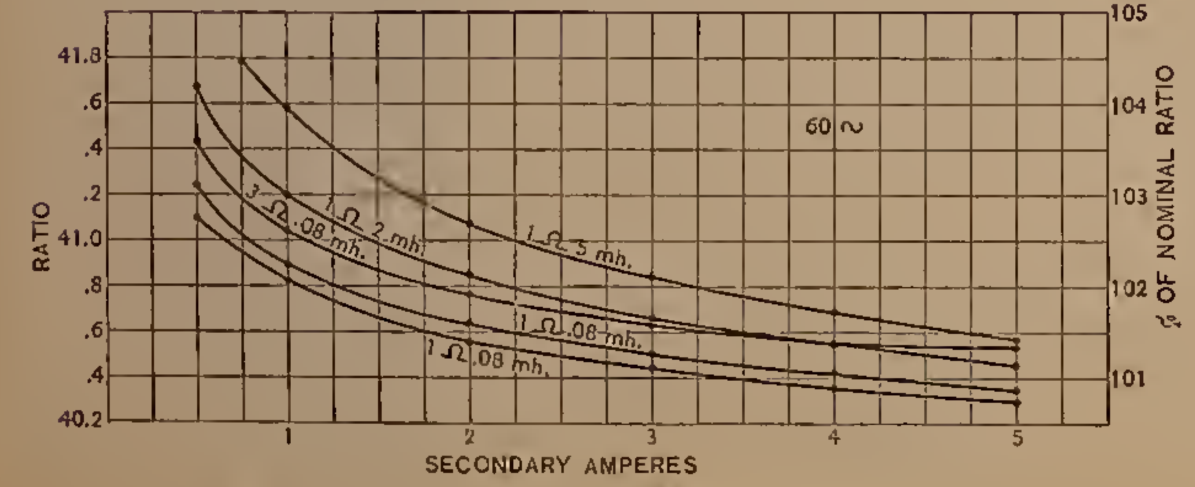


Fig. 14—*Ratio curves, current transformer B. 60 cycles.*

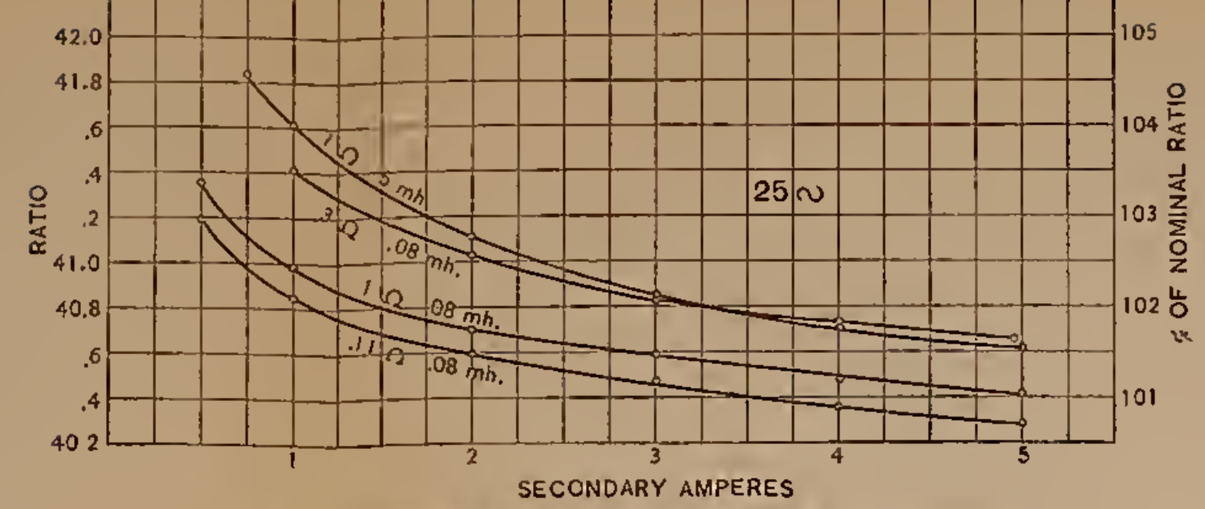


Fig. 15.—*Ratio curves, current transformer B. 25 cycles.*

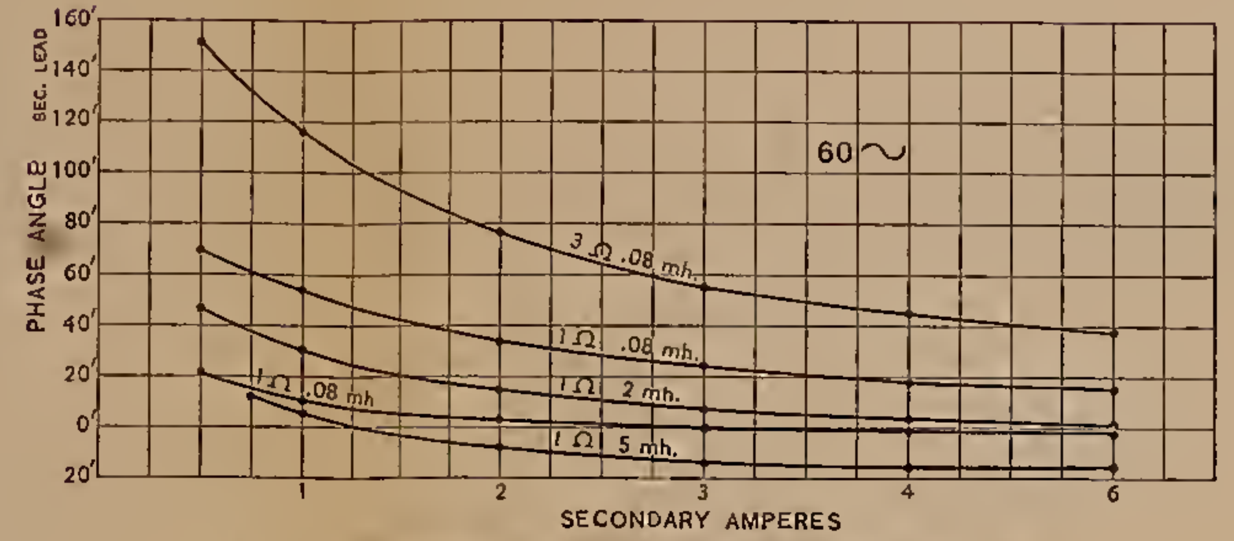


Fig. 16.—*Phase angle curves, current transformer B. 60 cycles.*

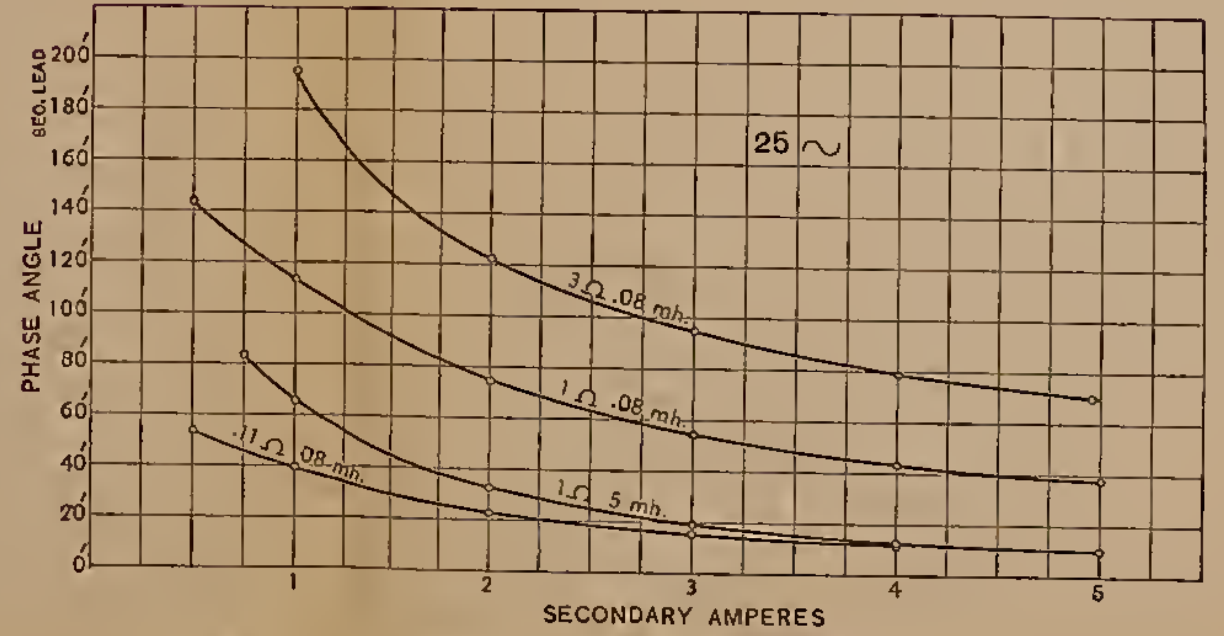


Fig. 17.—*Phase angle curves, current transformer B. 25 cycles.*

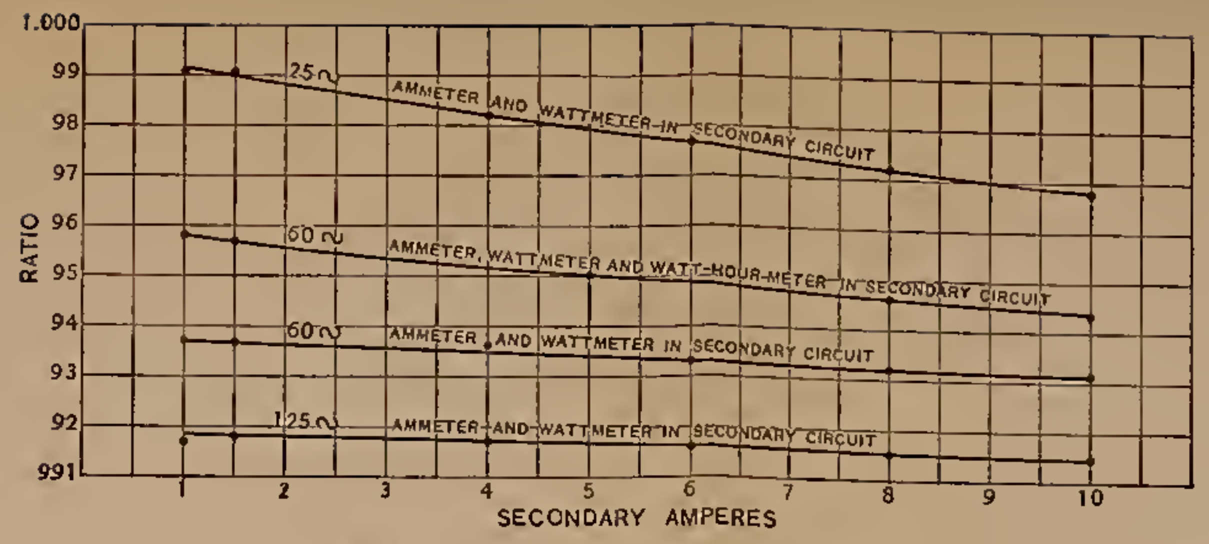


Fig. 18.—*Ratio curves, current transformer C.*

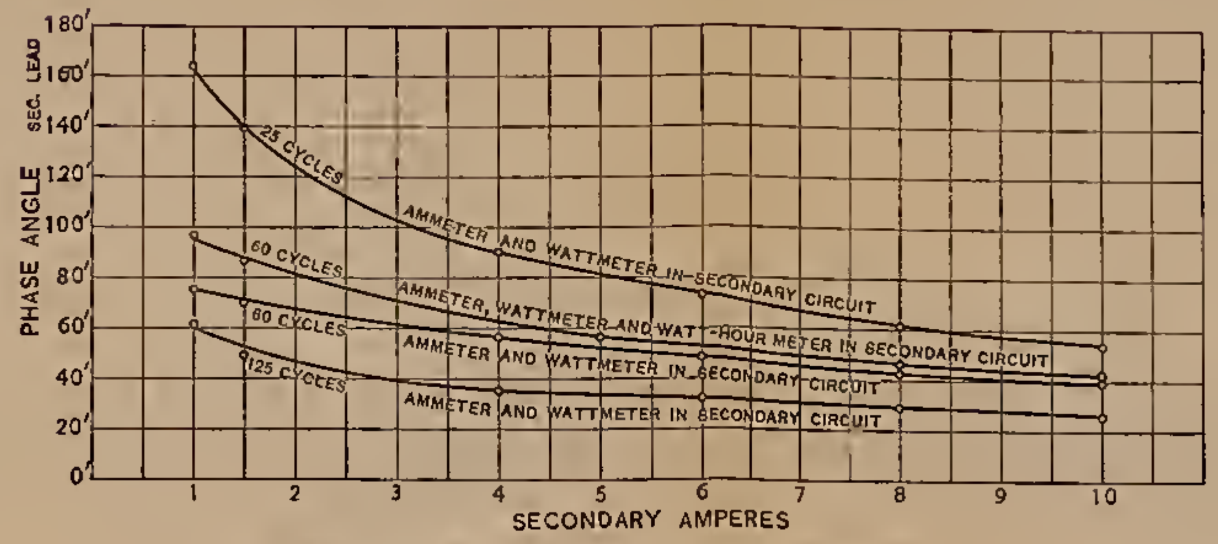


Fig. 19.—*Phase angle curves, current transformer C.*

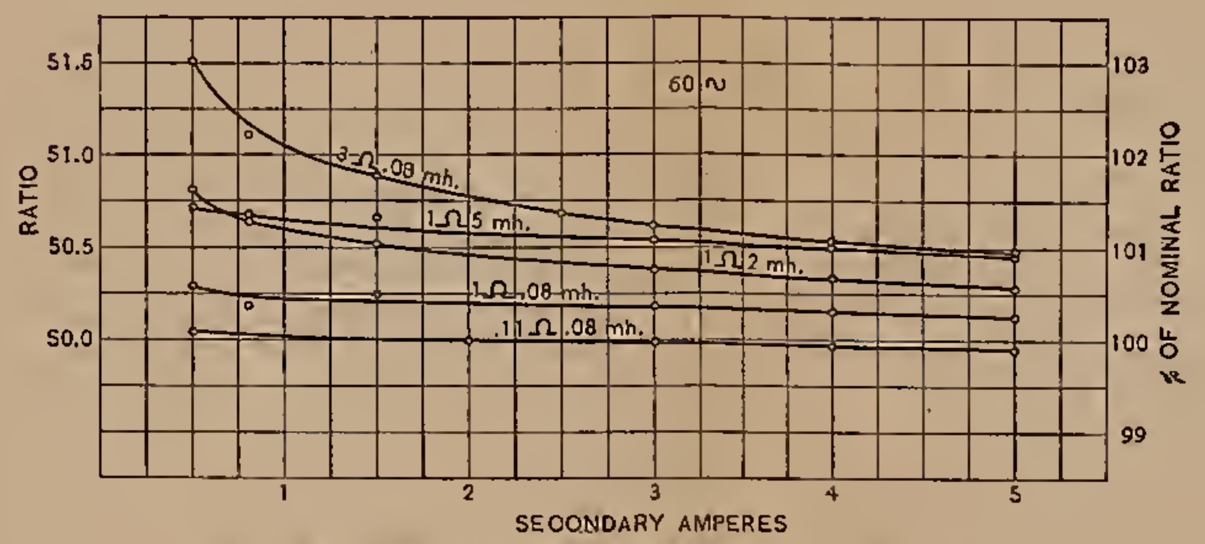


Fig. 20.—*Ratio curves, current transformer D. 60 cycles.*

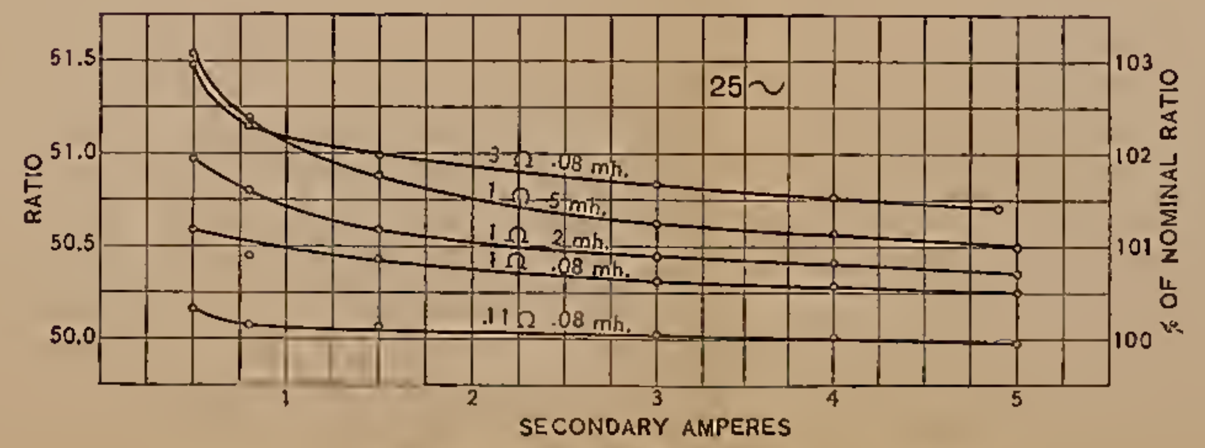


Fig. 21.—*Ratio curves, current transformer D. 25 cycles.*

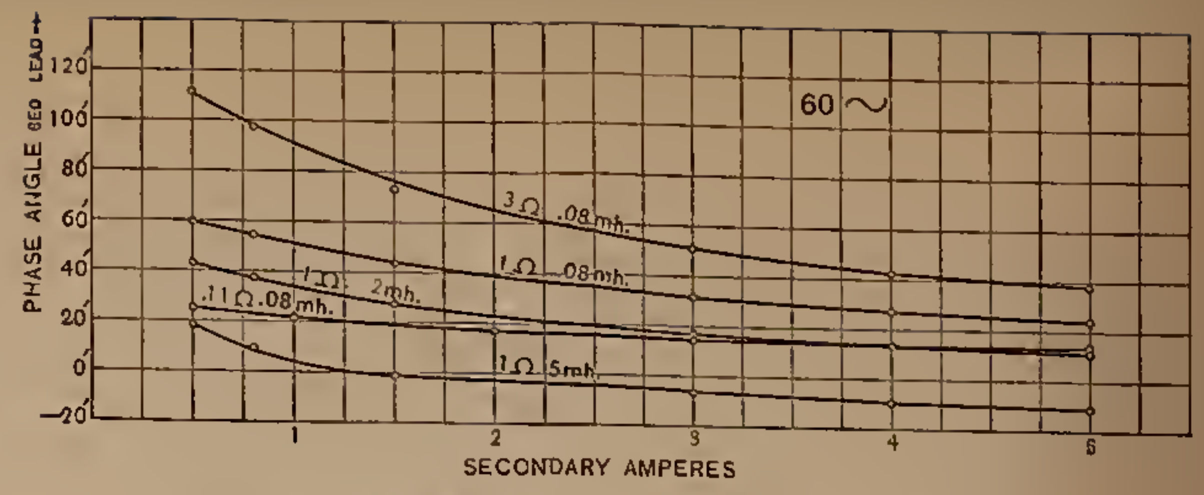


Fig. 22.—*Phase angle curves, current transformer D. 60 cycles.*

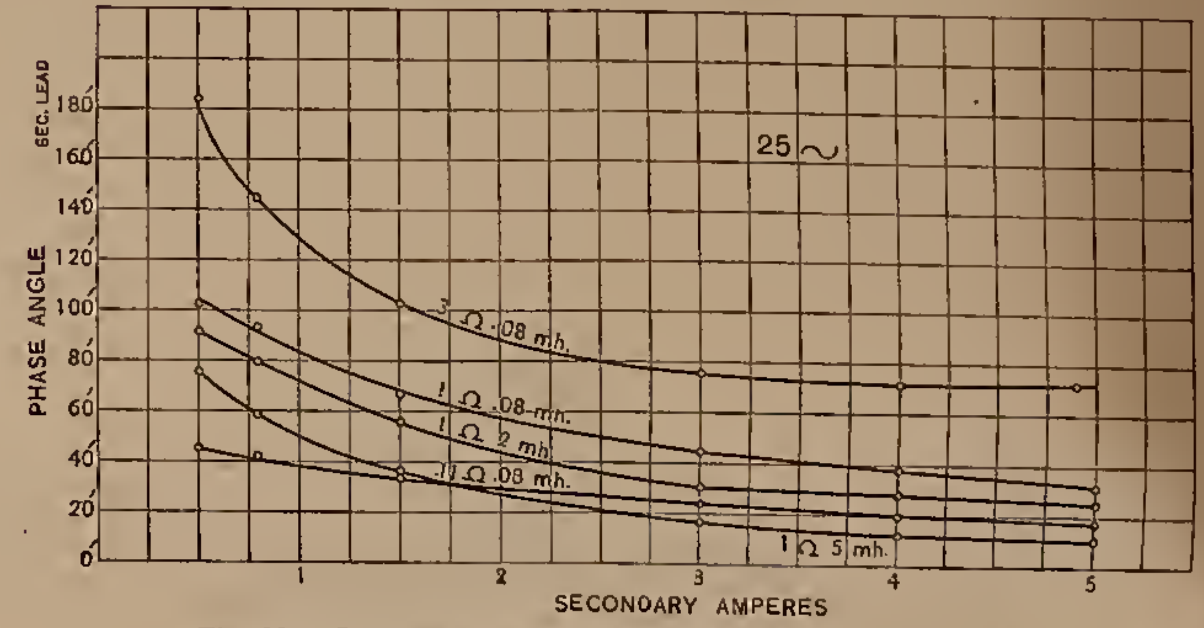


Fig. 23.—*Phase angle curves, current transformer D. 25 cycles.*

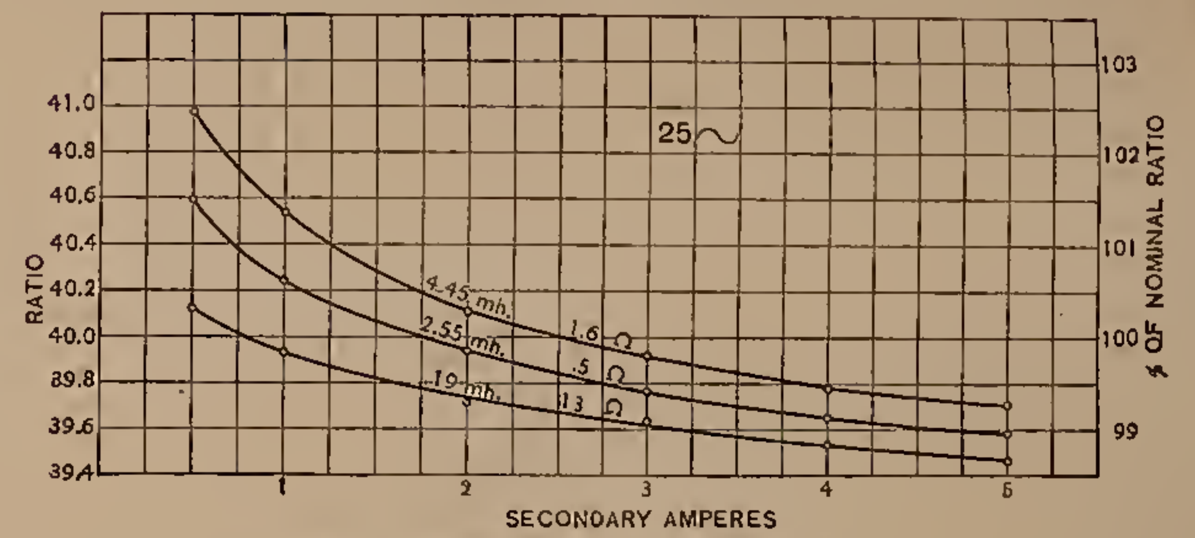


Fig. 24.—*Ratio curves, current transformer E.*

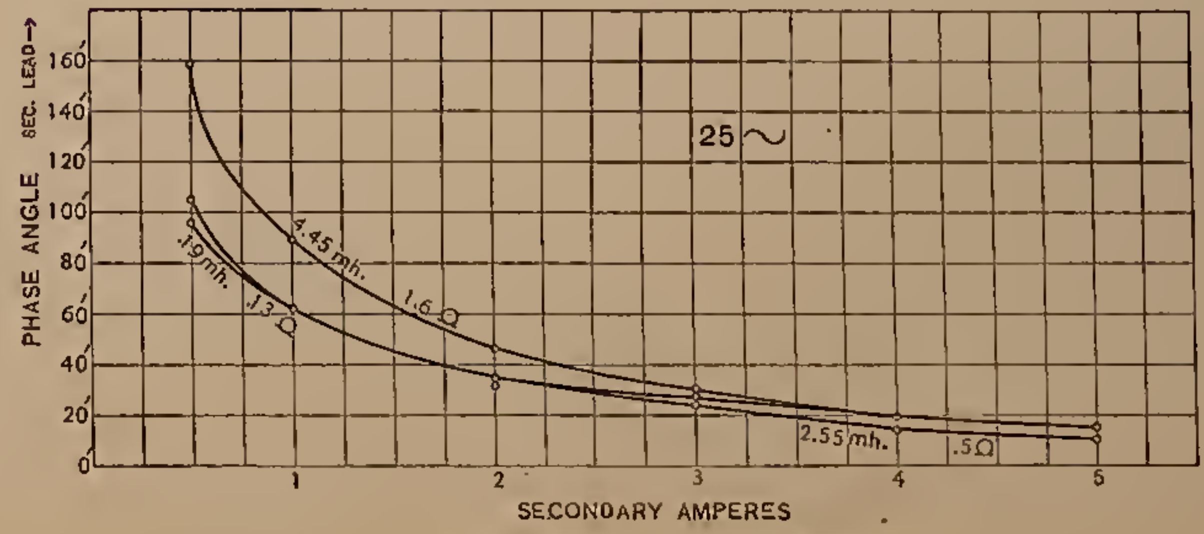


Fig. 25.—*Phase angle curves, current transformer E.*

rature. (Care must be taken that the deflection is in the proper direction, or the emf. may contain a resistance drop twice as great as the reactance drop, as well as the reactance drop. The two will be opposed so that the deflection may have the proper magnitude, but be in the wrong direction. One may make certain of having the correct connection by lagging the field current, when the deflection should decrease.) Now, if the moving coil be connected to the unknown shunt, the deflection measures the reactance emf. directly, or the deflections are proportional to the inductances and are independent of the resistances.

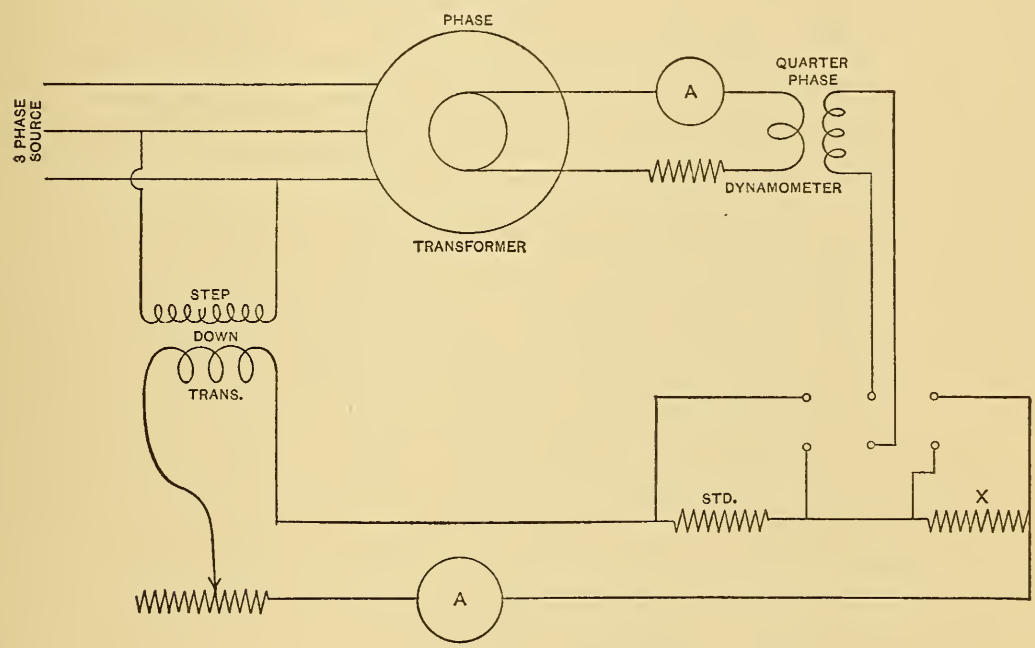


Fig. 28.—*Connections.*

As a check on the method, two shunts were constructed of forms that could be calculated, one concentric and one a flat shunt, doubled back upon itself. The comparison agreed to within 1 per cent of the calculated values. The values of the inductances compared were 3×10^{-8} and 126.8×10^{-9} henry.

Values of the phase angle of heavy low resistance "noninductive" shunts found by this method varied from 3 to 13 minutes at 60 cycles.[6]

WASHINGTON, July 5, 1909.

[6] We are indebted to Mr. H. B. Brooks for valuable suggestions during the course of the work, and to Mr. W. H. Stannard for assistance in taking readings and in the preparation of the data.